# A Fuzzy Two-Warehouse Inventory Model for Power Demand Pattern, Shortages with Partially Backlogged

## Wasim Akram Mandal, Sahidul Islam

Abstract: In this paper deals with fuzzy inventory model for non deteriorating item, power demand pattern, shortage under partially backlogged with two warehouse, is formulated and solved. After illustrate the model it test validity of the same, one numerical example have been solved then test sensitivity analyses. Fuzziness is applying by allowing the cost components (holding cost, shortage cost, etc). In fuzzy environment it considered all required parameter to be pentagonal fuzzy numbers. The purpose of the model is to minimize total cost function.

Keywords: Inventory, Two Ware-House, Power demand, Fuzzy number, Shortages, Pentagonal fuzzy number.

#### I. INTRODUCTION

An inventory deal with decision that minimum the total average cost or maximize The total average profit. For this purpose the task is to construct a mathematical model of the real life Inventory system, such a mathematical model is based on various assumption and approximation. In a inventory model shortages is very important condition. There are several type of customer. At shortage period some customers are waiting for actual product and others do not it. For this it consider partially- backlogging. When a retailer purchases a large quantity of goods at a time, then it hired one or more warehouse. In this paper it considered two warehouse OW and RW. Usually the holding cost in RW is higher than that in OW. The study of inventory model where demand rates varies with time is the last decades. Datta and pal investigated an inventory system with power demand pattern for item which variable rate deterioration. Park and Wang studied shortages and partial backlogging of items. Friedman(1978) presented continuous time inventory model with time varying demand. Ritchie(1984) studied in inventory model with linear increasing demand. Goswami, Chaudhuri(1991) discussed an inventory model with shortage. B.Das, and K.Maity(2008), a two warehouse supply-chain model. Gen et. Al. (1997) considered classical inventory model with Triangular fuzzy number .Yao and Lee(1998) considered an economic production quantity model in the fuzzy sense. Sujit Kumar De, P.K.Kundu and A.Goswami(2003) presented an economic production quantity inventory model involving fuzzy demand rate. J.K.Syde and L.A.Aziz(2007) applied sign distance method to fuzzy inventory model without shortage.

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Wasim Akram Mandal, Beldanga D. H. Sr. Madrasah, Beldanga-742189. Murshidabad. WB. India.

Sahidul Islam, Department of Mathematics, University of Kalyani, Kalyani, W.B, India.

M.K.Maity(2008), Fuzzy inventory with two warehouse under possibility measure of fuzzy goal. D.Datta and Pravin Kumar published several paper of fuzzy inventory with or without shortage. In ordinary inventory model it consider all parameter like shortage cost, holding cost, unit cost as fixed. But in real life situation it will have some little fluctuations . so consideration of fuzzy variables is more realistic. In this paper we first consider crisp inventory model with power demand where shortage are allowed and partially backlogged. Thereafter we developed fuzzy inventory model with fuzzy power demand rate under partially backlogged. All inventory cost parameters are fuzzyfied as pentagonal fuzzy number.

#### II. PRELIMINARIES

For graded representation method to defuzzyfy, we need the following definitions,

Definition 2.1: A fuzzy set  $\tilde{A}$  on the given universal set X is a set of order pairs,

 $\tilde{A} {=} \{(x, \mu_A(x)) \colon x {\in} X\} \ \text{ where } \ \mu_A(x) {\to} [0, 1] \ \text{ is called a membership function.}$ 

Definition2.2:The  $\alpha$ -cut of  $\tilde{A}$ , is defined by  $A_{\alpha} = \{x: \mu_A(x) = \alpha, \alpha \ge 0\}$ 

Definition 2.3:  $\tilde{A}$  is normal if there exists  $x \in X$  such that  $\mu_A(x)=1$ 

Definition 2.4: A pentagonal fuzzy number  $\tilde{A}$  = (a,b,c,d,e) is represented with membership function  $\tilde{A}$ 

 $\tilde{A}$  is defined as.

$$\mu_{A}(X) = \begin{cases} L_{1}(x) = \frac{x-a}{b-a}, a \leq x \leq b \\ L_{2}(x) = \frac{x-b}{c-b}, b \leq x \leq c \\ 1, \quad x = c \\ R_{1}(x) = \frac{d-x}{d-c}, c \leq x \leq d \\ R_{2}(x) = \frac{e-x}{e-d}, d \leq x \leq e \\ 0 \quad , otherwise \end{cases}$$

The  $\alpha$ -cut 0f  $\tilde{A}$ =(a,b,c,d,e),  $0 \le \alpha \le 1$  is  $A(\alpha)$ =[ $A_L(\alpha)$ , $A_R(\alpha)$ ]

Where  $A_{L_1}(\alpha)=a+(b-a)\alpha=L_1^{-1}(\alpha)$ 

 $A_{L_2}(\alpha) = b + (c-d)\alpha = L_2^{-1}(\alpha)$ And  $A_{R_1}(\alpha) = d - (d-c)\alpha = R_1^{-1}(\alpha)$ 

 $A_{R_1}(\alpha) = e - (e - d)\alpha = R_1^{-1}(\alpha)$  $A_{R_2}(\alpha) = e - (e - d)\alpha = R_2^{-1}(\alpha)$ 

So L<sup>-1</sup>( $\alpha$ )= $\frac{1}{2}$ [L<sub>1</sub><sup>-1</sup>( $\alpha$ )+L<sub>2</sub><sup>-1</sup>( $\alpha$ )]

 $= \frac{1}{2}[a+b+(c-a)\alpha]$   $R^{-1}(\alpha) = \frac{1}{2}[R_1^{-1}(\alpha) + R_2^{-1}(\alpha)]$ 

 $= \frac{1}{2}[d+e-(e-c)\alpha]$ 

Definition 2.5: If  $\tilde{A}$  = (a,b,c,d,e) is a pentagonal fuzzy number then the graded mean integration of



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$$\begin{split} \widetilde{A} \text{ is defined as,} \\ P(\widetilde{A}) &= \frac{\int_{0}^{W} A\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh}{\int_{0}^{W} A \, h \, dh}, (0 \leq h \leq W_{A} \text{ and } 0 \leq W_{A} \leq 1) \\ P(\widetilde{A}) &= \frac{\int_{0}^{1} \left[\frac{a + b + (c - a)h + d + e - (e - c)h\right] dh}{2}\right]}{\int_{0}^{1} h \, dh} \\ &= \frac{a + 3b + 4c + 3d + e}{12} \end{split}$$

#### **NOTATION** III.

 $I_r(t)$ :Inventory level at time t in RW, t $\geq 0$ .  $I_0(t)$ :Inventory level at time t in OW,  $t \ge 0$ . T:Cycle of length.

t<sub>w</sub>: Time point when stock level of RW reaches to zero. t<sub>1</sub>: Time point when stock level of OW reaches to zero.

c<sub>1</sub>:Fixed cost.

c<sub>2</sub>:Shortages cost per unit.

c<sub>3</sub>:Opportunity cost due to lost sales.

C<sub>w</sub>.c<sub>o</sub>:Holding cost per unit per unit time at RW and OW.

S:Highest stock level at the beginning of the cycle.

W:Stoarge capacity of OW.

R:Highest shortages level.

Q:Total order quantity per cycle.

 $TAC(t_w,t_1)$ :Total average cost per unit.

 $\tilde{c}_1$ =Fuzzy fixed cost.

 $\tilde{c}_2$ =Fuzzy shortage cost per unit.

 $\tilde{c}_3$ =Fuzzy opportunity cost due to lost sales.

 $\tilde{c}_{\mathrm{W}}, \tilde{c}_{\mathrm{0}}$  =Fuzzy holding cost per unit in RW, OW respectively, which is variable.

 $TA\widetilde{C(t_w, t_1)}$ =Fuzzy total cost per unit.

# 3.1. ASSUMTION:

a: The inventory system involves only one item.

b: The replenishment occur instantaneously at infinite rate.

c: The lead time is negligible.

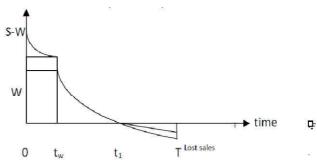
d: Demand rate is power demand, we assume it d(t/T)<sup>^1/n</sup> where d is constant. During the fixed

time T,  $\frac{dt^{(1-n)/n}}{nT^{1/n}}$  is demand rate at time t.

e: Backlogging rate is  $\frac{1}{1+\delta(T-t)}$ ,  $t_1 \le t \le T$ .

f: Holding cost variable as  $e^{\theta t}$ ,  $0 \le \theta < 1$ .

## 3.2. MODEL DEVOLOPMENT (CRISP SET)



Initially w units are store in OW and the rest (s-w) in RW. During 0≤t≤t<sub>w</sub>, the inventory (s-w) units in RW decrease due to customer demand and it vanishes at t=tw. during the time interval 0\leq t\leq t\_w, the inventory level remain same in OW and during the interval tw \( t \leq t \) it decrease due to customer demand and reaches to zero at t=t<sub>1</sub>. During the time interval t<sub>1</sub><t≤T, the partially backlogged shortages are allowed. So the differential equation describing as follows

$$\frac{dI_r(t)}{dt} = -\frac{dt^{\frac{1-n}{n}}}{nT^{1/n}}, 0 \le t \le t_w$$
 Subject to boundary condition  $I_r(t_w) = 0$ , and  $I_r(0) = S-W$ . (3.1)

$$\frac{dI_0(t)}{dt} = 0 , 0 \le t \le t_w$$
 (3.2)

to boundary condition 
$$I_r(t_w)=0$$
, and  $I_r(0)=S-w$ .

$$\frac{dI_0(t)}{dt}=0, 0 \le t \le t_w \qquad (3.2)$$
Subject to boundary condition  $I_0(t_w)=0$ .

$$\frac{dI_0(t)}{dt}=-\frac{dt^{\frac{1-n}{n}}}{nT^{1/n}}, t_w \le t \le t_1 \qquad (3.3)$$
Subject to boundary condition  $I_0(t_1)=0$ .

$$\frac{dI_0(t)}{dt}=-\frac{dt^{\frac{1-n}{n}}}{n\{1+\delta(T-T)\}T^{1/n}}, t_1 \le t \le T \qquad (3.4)$$
to boundary condition  $I_0(t_1)=0$ .

$$\frac{dI_0(t)}{dt} = -\frac{dt^{\frac{1-n}{n}}}{n\{1+\delta(T-T)\}T^{1/n}}, t_1 \le t \le T$$
 (3.4)

Subject to boundary condition  $I_0(t_1)=0$ . And  $I_0(T)=-R$ 

From (3.1) we get,

$$I_{r}(t) = \frac{d}{T^{1/n}} \left( t_{W}^{1/n} - t^{1/n} \right)$$
 (3.5)

So S-W=  $\frac{d}{T^{1/n}} t_W^{1/n}$ 

From (3.2) we get,

$$I_0(t) = W \tag{3.6}$$

From (3.3) we get,

$$I_0(t) = \frac{d}{T^{1/n}} \left( t_1^{1/n} - t^{1/n} \right)$$
 (3.7)

From (3.4) we get,  

$$I_0(t) = \frac{d}{T^{1/n}} [(t^{1/n} - t_1^{1/n})(1 - \delta T) + \frac{\delta}{1+n} \{t^{(1+n)/n} - t_1^{(1+n)/n}\}]$$
(3.8)

So the maximum stock level at the beginning of the cycle is,  

$$S=W + \frac{d}{T^{1/n}} t_{w}^{1/n}$$
(3.9)

And the maximum amount of demand backlogging is,

$$R=-I_{0}(T)$$

$$=\frac{d}{T^{1/n}}[(T^{1/n}-t_{1}^{1/n})(1-\delta t)+\frac{\delta}{1+n}\{T^{(1+n)/n}-t_{1}^{(1+n)/n}\}] \quad (3.10)$$

Hench the order quantity per cycle is;

Q=S+R

$$=W + \frac{d}{T^{1/n}} \left[ t_{w}^{1/n} + (T^{1/n} - t_{1}^{1/n})(1 - \delta T) + \frac{\delta}{1 + n} \left\{ T^{(1 + n)/n} - t_{1}^{(1 + n)/n} \right\} \right]$$

The fixed cost per cycle is,

 $FC=c_1$ 

Shortages cost per cycle is,

$$Sc=-c_2\int_{t_1}^T I(t)dt,$$

$$=-c_{2}\frac{d}{T^{1/n}}[(1-\delta t)(T-t_{1})t_{1}^{1/n}-\frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n}t_{1}^{(n+1)/n}\}+\frac{\delta}{1+n}t_{1}^{(n+1)/n}(T-T_{1})-\frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n}-t_{1}^{(2n+1)}\}]$$

Opportunity cost due to lost sales is,

OC=
$$c_3 \int_{t1}^{T} R(t) \left[1 - \frac{1}{1 + \delta(T - t)}\right] dt$$
  
= $c_3 \frac{\delta d}{T^{1/n}} \left[T(T^{1/n} - t_1^{1/n}) - \frac{1}{n+1} \left\{T^{(n+1)/n} - t_1^{(n+1)/n}\right\}\right].$ 

Holding cost per cycle is,



As  $\theta$ , is too small, so neglecting higher power of  $\theta$ .

$$\begin{split} & \text{HC=c}_{\mathbf{w}} \int_{0}^{t_{w}} I_{r}(t) \mathrm{e}^{\theta t} dt + c_{0} \int_{0}^{t_{w}} I_{0}(t) \mathrm{e}^{\theta t} dt + c_{0} \int_{t_{w}}^{t_{1}} I_{r}(t) \mathrm{e}^{\theta t} dt \\ &= \frac{d}{T^{n}} [c_{\mathbf{w}} (\frac{t_{w}^{\frac{1+n}{n}}}{1+n} + \theta \frac{t_{w}^{\frac{1+2n}{n}}}{2+4n}) + c_{0} (t_{\mathbf{w}} + \frac{\theta}{2} t_{w}^{2}) + c_{0} (\frac{t_{1}^{\frac{1+n}{n}}}{1+n} + \theta \frac{t_{1}^{\frac{1+2n}{n}}}{2+4n} - t_{\mathbf{w}} (t_{1}^{\frac{1}{n}} - \frac{n}{1+n} t_{w}^{\frac{1}{n}}) - t_{w}^{2} (\frac{t_{1}^{\frac{1}{n}}}{2} - \frac{n}{1+2n} t_{w}^{\frac{1}{n}})]. \end{split}$$

$$TAC(t_{w},t_{1}) = \frac{1}{T}[FC+SC+OC+HC]$$

$$= \frac{d}{T^{\frac{1+n}{n}}} \Big[ c_1 T^{\frac{1}{n}} - c_2 \Big[ (1-\delta t)(T-t_1) t_1^{1/n} - \frac{n(1-\delta T)}{n+1} \Big\{ T^{(n+1)/n} - t_1^{(n+1)/n} \Big\} + \frac{\delta}{1+n} t_1^{(n+1)/n} (T-T_1) - \frac{n\delta}{(n+1)(2n+1)} \Big\{ T^{(2n+1)/n} - t_1^{(2n+1)} \Big\} \Big] + c_3 \quad \Big[ T(T^{1/n} - t_1^{1/n}) - \frac{1}{n+1} \Big\{ T^{(n+1)/n} - t_1^{(n+1)/n} - t_1^{(n+1)/n} \Big\} \Big] + \Big[ c_w \Big( \frac{t_w}{1+n} + \theta \frac{t_w}{2+4n} \Big) + c_0 \Big( t_w + \frac{\theta}{2} t_w^2 \Big) + c_0 \Big\{ \frac{t_1}{1+n} + \theta \frac{t_1+2n}{2+4n} - t_w \Big( t_1^{\frac{1}{n}} - \frac{n}{1+n} t_w^{\frac{1}{n}} \Big) - t_w^2 \Big( \frac{t_1^{\frac{1}{n}}}{2} - \frac{n}{1+2n} t_w^{\frac{1}{n}} \Big) \Big\} \Big].$$

For minimum cost it should be

$$\frac{\partial TAC(t_w, t_1)}{\partial t_w} = 0, \quad \frac{\partial TAC(t_w, t_1)}{\partial t_1} = 0$$

Provided it satisfies equation,

$$\begin{split} \frac{\partial^2 \text{TAC}(\mathsf{t}_w, \mathsf{t}_1)}{\partial t_w^2} > &0, \quad \frac{\partial^2 \text{TAC}(\mathsf{t}_w, \mathsf{t}_1)}{\partial t_1^2} > &0\\ \text{And} \quad \left[\frac{\partial^2 \text{TAC}(\mathsf{t}_w, \mathsf{t}_1)}{\partial t_w^2}\right] \left[\frac{\partial^2 \text{TAC}(\mathsf{t}_w, \mathsf{t}_1)}{\partial t_w^2}\right] - \left[\frac{\partial^2 \text{TAC}(\mathsf{t}_w, \mathsf{t}_1)}{\partial t_w \partial t_1}\right] > &0. \end{split}$$

#### 3.3 FUZZY MODEL:

Due to uncertainly lets us assume that,

$$\widetilde{c}_{1} = (c_{1}^{1}, c_{1}^{2}, c_{1}^{3}, c_{1}^{4}, c_{1}^{5}), \qquad \widetilde{c}_{2} = (c_{2}^{1}, c_{2}^{2}, c_{2}^{3}, c_{2}^{4}, c_{2}^{5}), \quad \widetilde{c}_{3} = (c_{3}^{1}, c_{3}^{2}, c_{3}^{3}, c_{3}^{4}, c_{3}^{5}), \quad \widetilde{c}_{W} = (c_{w}^{1}, c_{w}^{2}, c_{w}^{3}, c_{w}^{4}, c_{w}^{5}), \quad \widetilde{c}_{0} = (c_{0}^{1}, c_{0}^{2}, c_{0}^{3}, c_{0}^{4}, c_{0}^{5}), \text{ be pentagonal fuzzy number then the total average cost is given by,}$$

$$TA\widetilde{C(t_w, t_1)} = \frac{1}{\pi} [FC + SC + OC + HC]$$

$$= \frac{d}{T^{\frac{1+n}{n}}} [\widetilde{c_1} T^{\frac{1}{n}} - \widetilde{c_2} [(1-\delta t)(T-t_1)t_1^{1/n} - \frac{n(1-\delta T)}{n+1} \{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{\delta}{1+n} t_1^{(n+1)/n} (T-T_1) - \frac{n\delta}{(n+1)(2n+1)} \{T^{(2n+1)/n} - t_1^{(2n+1)}\}] + \widetilde{c_3} [T(T^{1/n} - t_1^{1/n}) - \frac{1}{n+1} \{T^{(n+1)/n} - t_1^{(n+1)/n}\}] + [\widetilde{c_w} (\frac{t_w}{n}) + \frac{t_1^{\frac{1+2n}{n}}}{2+4n}) + \widetilde{c_0} (t_w + \frac{\theta}{2} t_w^2) + \widetilde{c_0} \{ \frac{t_1^{\frac{1+n}{n}}}{1+n} + \frac{t_1^{\frac{1+2n}{n}}}{2+4n} - t_w(t_1^{\frac{1}{n}} - \frac{n}{1+n} t_w^{\frac{1}{n}}) - t_w^2(\frac{t_1^{\frac{1}{n}}}{2+4n} - t_w^2) + \frac{t_1^{\frac{1}{n}}}{2+4n} - t_w^2(t_1^{\frac{1}{n}} - \frac{n}{1+n} t_w^{\frac{1}{n}}) - t_w^2(\frac{t_1^{\frac{1}{n}}}{2+4n} - t_w^2) + t_w^2(t_1^{\frac{1}{n}} - \frac{n}{1+2n} t_w^{\frac{1}{n}}) \}].$$

We defuzzyfi the fuzzy total cost  $\widetilde{TAC}(t_1)$  by graded mean representation method as follows,

$$\begin{split} TA\widetilde{C(t_{w}},t_{1}) = & \frac{1}{12}[\widetilde{TAC^{1}}(t_{w},t_{1}),\widetilde{TAC^{2}}(t_{w},t_{1}),\widetilde{TAC^{3}}(t_{w},t_{1}),\widetilde{TAC^{4}}(t_{w},t_{1}),\widetilde{TAC^{5}}(t_{w},t_{1})] \\ \widetilde{TAC^{r}}(t_{1}) = & \frac{d}{\frac{1+n}{r}}[\widetilde{c^{r}}_{1}T^{\frac{1}{n}} - \widetilde{c^{r}}_{2} - [(1-\delta t)(T-t_{1})t_{1}^{-1/n} - \frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n} - t_{1}^{-(n+1)/n}\} + \frac{\delta}{1+n}t_{1}^{-(n+1)/n}(T-T_{1}) - \frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n} - t_{1}^{-(2n+1)/n} - t_{1}^{-(2n+1)/n}\} + \widetilde{c^{r}}_{3} - [T(T^{1/n} - t_{1}^{-1/n}) - \frac{1}{n+1}\{T^{(n+1)/n} - t_{1}^{-(n+1)/n}\} + [\widetilde{c^{r}}_{w}(\frac{t_{w}}{n}) + \frac{t_{w}}{n} + \theta \frac{t_{w}}{n} - t_{w}}{\frac{t_{w}}{n}} + \theta \frac{t_{w}}{n} + \theta \frac{t_{w}}{n} + \theta \frac{t_{w}}{n} - t_{w}}{\frac{t_{w}}{n}} + \theta \frac{t_{w}}{n} + \theta \frac{t_{w}}{n} - t_{w}}{\frac{t_{w}}{n}} + \theta \frac{t_{w}}{n} - t_{w}} + \widetilde{c^{r}}_{0}(t_{w} - t_{w}) + \widetilde{c^$$

For minimum cost it should be,
$$\frac{\partial TAC(\widetilde{t_w},t_1)}{\partial t_w} = 0, \quad \frac{\partial TAC(\widetilde{t_w},t_1)}{\partial t_1} = 0$$

Provided it satisfies equation

$$\begin{split} \frac{\partial^2 TAC(t_w,t_1)}{\partial t_w} > &0, \quad \frac{\partial^2 TAC(t_w,t_1)}{\partial t_w} > 0 \\ \text{And} \quad \left[ \frac{\partial^2 TAC(t_w,t_1)}{\partial t_w} \right] \left[ \frac{\partial^2 TAC(t_w,t_1)}{\partial t_w} \right] - \left[ \frac{\partial^2 TAC(t_w,t_1)}{\partial t_w \partial t_1} \right] > &0. \end{split}$$

#### NUMERICAL SOLUTION

For crisp model: Let us take the in-put value:

Tot erisp model. Let us take the in put value.										
$C_1$	$C_2$	$C_3$	$C_{\mathrm{w}}$	$C_0$	δ	θ	n	d	T	W
100	5	10	10	6	0.1	0.2	3	50	2	100

And the out-put value:

$t_{\rm w}$	$t_1$	Q	$TAC(t_{w,}t_1)$
0.132	0.432	138.382	2361.946



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 $t_1 = 0.431$ 

For fuzzy model:

 $\widetilde{c_1}$ =(90,95,100,105,110),  $\widetilde{c_2}$  =(3,4,5,6,7),  $\widetilde{c_3}$ =(6,8,10,12,14),

 $\widetilde{c_w} = (8,9,10,11,12), \ \widetilde{c_0} = (4,5,6,7,8)$ 

The solution of fuzzy model by graded mean representation is,

(1) When  $\tilde{c_1}, \tilde{c_2}, \tilde{c_3}, \tilde{c_w}, \tilde{c_0}$  are all pentagonal fuzzy numbers then.

 $TAC(t_w,t_1)=2349.253$ ,  $t_w=0.131$ 

(2) When  $\widetilde{c_1},\widetilde{c_2},\widetilde{c_3},\widetilde{c_w}$ , are all pentagonal fuzzy numbers then

 $TAC(t_w,t_1)=2348.423, t_w=0.133 t_1=0.434$ 

(3) When  $\widetilde{c_1}, \widetilde{c_2}, \widetilde{c_3}$ , are pentagonal fuzzy numbers then,

 $TAC(t_w,t_1)=2347.936, t_w=0.128 t_1=0.430$ 

(4) When  $\widetilde{c_1}$ ,  $\widetilde{c_2}$ , are pentagonal fuzzy numbers then,

 $TAC(t_w,t_1)=2361.915, t_w=0.132$   $t_1=0..432$ 

(5) When  $\widetilde{c}_1$ , are pentagonal fuzzy numbers then,

TAC( $t_w.t_1$ )=2359.645,  $t_w$ =0.1132  $t_1$ =0.432

#### V. SENSITIVITY ANALYSIS(CRISP MODEL)

We now examine to sensitivity analysis of the optimal solution of the model for change in I, keeping the other parameters unchanged. The initial data from the above numerical example.

Parameter	% of change	$TAC(t_w,t_1)$	$t_{\rm w}$	$t_1$
$C_1 = 50.0$	-50	1111.946	0.132	0.432
$C_1 = 75.0$	-25	1736.946	0.132	0.432
$C_1 = 100$	0	2361.946	0.132	0.432
$C_1 = 125$	25	2986.946	0.132	0.432
$C_1 = 150$	50	3611.946	0.132	0.432
$C_2=2.50$	-50	2430.973	0.132	0.432
$C_2=3.75$	-25	2396.460	0.132	0.432
$C_2 = 5.00$	0	2361.946	0.132	0.432
$C_2=6.25$	25	2327.433	0.132	0.432
$C_2 = 7.50$	50	2292.420	0.132	0.432
$C_3=5.00$	-50	2508.623	0.120	0.399
$C_3 = 7.50$	-25	2435.385	0.128	0.429
$C_3=10.0$	0	2361.946	0.132	0.432
$C_3=12.5$	25	2288.394	0.136	0.442
$C_3=15.0$	50	2214.773	0.138	0.449
$C_W = 5.00$	-50	2348.168	0.276	0.492
$C_W = 7.50$	-25	2356.709	0.187	0.454
$C_{\rm w} = 10.0$	0	2361.946	0.132	0.432
$C_W = 12.5$	25	2365.342	0.097	0.419
$C_W = 15.0$	50	2367.649	0.074	0.410
$C_0 = 3.00$	-50	2347.892	0.056	0.480
$C_0=4.50$	-25	2356.262	0.097	0.451
$C_0 = 6.00$	0	2361.946	0.132	0.432
$C_0 = 7.50$	25	2365.884	0.161	0.421
$C_0 = 9.00$	50	2368.659	0.186	0.414

## 5.2(a)Effect, for increment parameters-

- (1)  $TAC(t_w,t_1)$  increase, for increase of  $c_1$ .
- (2)  $TAC(t_w,t_1)$  decrease, for increase of  $c_2$ .
- (3)  $TAC(t_w,t_1)$  decrease, for increase of  $c_3$ .
- (4)  $TAC(t_w,t_1)$  increase, for increase of  $c_w$ .
- (5)  $TAC(t_w,t_1)$  increase, for increase of  $c_0$ .

# VI. CONCLUSON

In this paper, we have proposed a real life two warehouse inventory problem in a fuzzy environment and presented solution along with sensitivity analysis approach. The inventory model developed with power pattern demand with shortages. Shortages have been allow partially backlogged in this model. In case where large portion of demand occurs at the beginning of the period the author, use n>1 and when it is large at end we use 0<n<1. This model has been developed for single item. In this paper, we have considered pentagonal fuzzy number and solved by graded mean integration method. In future, the other type of

membership functions such as piecewise linear hyperbolic, L-R fuzzy number, etc can be considered to construct the membership function and then model can be easily solved.

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