Vibration Analysis of Thick Plate by Using Refined Plate Theory and ANSYS

I. I. Sayyad, S. M. Hon, K. K. Joshi, P. N. Kolase, Omkar Babasaheb Kale

Abstract— ***refined plate theory is applied for free vibration analysis of thick plate for better results and greater accuracy. In this paper vibration analysis of thick isotropic plate is carried out and results are compared with the results of ANSYS APDL (14.5). This theory uses sinusoidal function in terms of thickness coordinate and accounts for realistic variation of the transverse shear stress through the thickness and satisfies the shear stress free surface conditions at the top and bottom surfaces of the plate. Simply supported thick isotropic plate is considered for detail numerical study. Navier's solution technique is used for the analytical solution. The results are obtained for natural bending mode frequencies. ANSYS APDL 14.5 is used to obtain fundamental frequencies in Modal Solution.***

Keywords: Natural Frequencies, ANSYS, Shear Correction Factor, Shear Deformation, Transverse Shear Stress, Modal analysis.

I. INTRODUCTION

For the analysis of plate we use various theories. Mainly these are,

1. Classical Plate Theory (CPT),
2. First Order Shear Deformation Theory (FSDT),
3. Higher order Shear Deformation Theories (HSDT).
4. Trigonometric Shear Deformation Theory (TSDT).

In classical plate theory, it is assumed that line which is normal to the neutral surface before deformation remain straight and normal to the neutral surface after deformation. This assumption results in under-estimation of deflection and over-estimation of natural frequencies. [1-3] First Order Shear Deformation Theory (FSDT) [4] is the starting point in the development of plate theories in which transverse shear effects were included. In this theory, constant shear strain is assumed across the thickness and it predicts average shear stress. It also requires a shear correction coefficient, accurate evaluation of which is problem specific. This theory has been widely used for static, free vibration and transient analysis because of its simplicity and good global predictions. The plate theories are further refined by assuming parabolic (higher order parabolas) shear strain variation across the thickness and these are called as Higher order Shear Deformation Theories (HSDT).

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It is very well known that HSDT gives more accurate results and very close to three dimensional (3-D) elasticity solution for static loading conditions and free vibrations. Higher order theories are based on realistic displacement models, Which give rise to nonlinear distribution of in-plane, normal and transverse shear strains. The higher order theories are quite involved and are more complicated as compared to the CPT and FSDT. In the present work, emphasis has been laid on specific development of Trigonometric Shear Deformation Theory (TSDT) for plate analysis. And the effectiveness of this theory is shown by applying it to static and dynamic problems of isotropic plates. Higher order plate theories are being used for the analysis proposed by K. H. Lo, R.M. Christensen and E.M. Wu [6]. Use of trigonometric functions to describe the plate behavior in thickness direction was first proposed by Stein [7] and was used for laminated beam and post buckling analysis of plates. The objective of this paper is to present a trigonometric shear deformation theory for isotropic thick plates. It includes the effect of transverse shear. Results obtained for uniformly distributed loading case and are compared with those of refined theories like J.N. Reddy, and N.D. Phan [8], Ghugal and Sayyad[9], Krishnamurthy[10], Reddy[11], Mindlin[3], Kirchhoff [2] and exact elasticity theory available in the literature. Also ANSYS is leading software for the purpose of analyzing any kind of system. [12-13] Also the great work is done by Sayyad and Chikhalthankar for Refined Plate Theory. [14] Now a day number of softwares are developed to find out the solution for the problems. These software mainly uses the Finite Element Method to find out the solutions. ANSYS is the quite familiar. In this paper we carried out the modal analysis of isotropic plate with the plates as simple supported boundary condition using the Finite Element Method in ANSYS. We have studied all results which are obtained by previous theory for their complications and for their accuracy. After that we choose the Refined Plate Theory and compared those results with Results obtained by ANSYS.

II. Theoretical Formulation

A. The Plate Under Consideration:

The plate under consideration occupies in O – x – y - z Cartesian coordinate system the region:

$$0 \le x \le a \; ; \qquad 0 \le y \le b \; ; \quad -h/2 \le z \le h/2 \qquad \ldots(1)$$

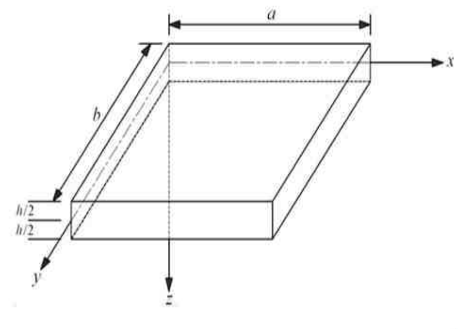

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Fig. 1: Geometry of Thick plate

Where.

a=100mm,

b=100mm,

h=10mm,

x, y, z are Cartesian co-ordinates, *a* and *b* are the edge lengths in the x and y directions respectively, and h is the thickness of the plate.

Assumptions Made In The Theoretical Formulation:

Assumptions are made for this theory are as same as that of presented by I. I. Sayyad.[14]

B. The Displacement Field:

Based on the before mentioned assumptions, the displacement field of the Present Refined Plate Theory (RPT) can be expressed as follows:

$$u = -z\frac{\partial w}{\partial x} + \frac{h}{\pi}\sin\frac{\pi z}{h}\ \psi(x,y,t)$$

$$v = -z\frac{\partial w}{\partial y} + \frac{h}{\pi}\sin\frac{\pi z}{h}\ \psi(x,y,t)$$

$$w = w\,(x, y, t) \qquad \ldots (2)$$

Here *u* and *v* are the in plane displacement components in the *x* and *y* directions respectively and *w* is the transverse displacement in the z direction. The trigonometric function in terms of thickness coordinates in both the displacements *u* and *v* is associated with the transverse shear stress distribution through the thickness of plate and the functions $\phi\,(x, y, t)$ and $\psi(x, y, t)$ are the unknown functions associated with the shear slopes.

***C. Strain-Displacement Relationships*:**

Normal and shear strains are obtained within the framework of linear theory of elasticity using the displacement field given by (2). These relationships are given as follows:

Normal Strain:

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial^2 x} + \frac{h}{\pi}\sin\frac{\pi z}{h}\frac{\partial \emptyset}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial^2 y} + \frac{h}{\pi}\sin\frac{\pi z}{h}\frac{\partial \psi}{\partial y}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = 0 \qquad \ldots (3)$$

Shear Strains:

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2\frac{\partial^2 w}{\partial x\,\partial y} + \frac{h}{\pi}\sin\frac{\pi z}{h}\left(\frac{\partial \emptyset}{\partial y} + \frac{\partial \psi}{\partial x}\right)$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} = \cos\frac{\pi z}{h}\ \emptyset$$

$$\gamma_{yz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = \cos\frac{\pi z}{h}\ \psi \qquad \ldots (4)$$

Stress-Strain Relationships:

For a linearly elastic isotropic material, stresses τ_{xy}, σ_x and σ_y are related to strains γ_{xy}, ε_x and ε_y shear stresses are related to shear strains by the following constitutive relations:

$$\begin{Bmatrix}\sigma x\\ \sigma y\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}1 & \mu & 0\\ \mu & 1 & 0\\ 0 & 0 & \frac{1-\mu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma xy\end{Bmatrix}$$

$$Q_{11}=Q_{12}=Q_{22}$$

$$Q_{44}=Q_{55}=Q_{66}=G \qquad \ldots. (5)$$

Where,

σ_x, σ_y are the in plane stresses,

$(\varepsilon x, \varepsilon y)$ are normal strain component,

$(\gamma xy\ , \gamma yz\ , \gamma zx)$ are the shear strain components and

Qij are the stiffness coefficients [27].

D. Governing Equations and Boundary Conditions:

Using the expressions for strains and stresses through (5) and using the principle of virtual work, variationally consistent governing differential equations and boundary conditions for the plate under consideration can be obtained.

The principle of virtual work when applied to the plate leads to:

In the principal of virtual work the main assumption used is Internal Workdone = External Workdone. We have applied external workdone as a load *q(x, y).*

$$\int_{z=-\frac{h}{2}}^{z=\frac{h}{2}}\int_{y=-b}^{y=b}\int_{x=-a}^{x=a}\begin{bmatrix}\sigma x\ \delta\varepsilon_x + \sigma y\ \delta\varepsilon_y + \sigma z\ \delta\varepsilon_z\\ \tau_{yz}\delta\gamma_{yz} + \tau_{zx}\delta\gamma_{zx} + \tau_{xy}\delta\gamma_{xy}\end{bmatrix}dxdydz - \int_{y=-b}^{y=b}\int_{x=-a}^{x=a} q(x,y)\ \delta w\ dx\ dy + \rho\int_{z=-h/2}^{z=h/2}\int_{y=-b}^{y=b}\int_{x=-a}^{x=a}\left[\frac{\partial^2 u}{\partial t^2}\delta u + \frac{\partial^2 v}{\partial t^2}\delta v + \frac{\partial^2 w}{\partial t^2}\delta w\right]dxdydz = 0 \qquad \ldots (6)$$

Where,

δ denotes the variation operator.

Integrating (6) by parts and collecting coefficients of δw, $\delta\phi$ and $\delta\psi$ following governing differential equations and the

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associated boundary conditions are obtained.

The governing differential equations obtained are as follows:

$$\left\{\begin{matrix} Q_{11}A\frac{\partial^4 w}{\partial x^4}+(2Q_{12}A+4Q_{66}A)\frac{\partial^4 w}{\partial x^2\partial y^2}+Q_{22}A\frac{\partial^4 w}{\partial y^4} \\ -Q_{11}B\frac{\partial^3 \emptyset}{\partial x^3}-(Q_{12}B+2Q_{66}B)\frac{\partial^3 \emptyset}{\partial x\,\partial y^2} \\ -Q_{22}B\frac{\partial^3 \psi}{\partial y^3}-(Q_{12}B+2Q_{66}B)\frac{\partial^3 \psi}{\partial x^2\partial y}-\rho A\frac{\partial^4 w}{\partial x^2\partial t^2} \\ -\rho A\frac{\partial^4 w}{\partial y^2\partial t^2}+\rho B\frac{\partial^3 \emptyset}{\partial x\,\partial t^2}+\rho B\frac{\partial^3 \psi}{\partial y\,\partial t^2}+\rho H\frac{\partial^2 w}{\partial t^2} \end{matrix}\right\} = q$$

$$\left\{\begin{matrix} Q_{11}B\frac{\partial^3 w}{\partial x^3}+(Q_{12}B+2Q_{66}B)\frac{\partial^3 w}{\partial x\,\partial y^2}-Q_{11}C\frac{\partial^2 \emptyset}{\partial x^2} \\ -Q_{66}C\frac{\partial^2 \emptyset}{\partial y^2}-(Q_{12}C+Q_{66}C)\frac{\partial^2 \psi}{\partial x\,\partial y}+Q_{55}D\emptyset \\ -\rho B\frac{\partial^3 w}{\partial x\,\partial t^2}+\rho C\frac{\partial^2 \emptyset}{\partial t^2} \end{matrix}\right\} = 0$$

$$\left\{\begin{matrix} Q_{22}B\frac{\partial^3 w}{\partial y^3}+(Q_{12}B+2Q_{66}B)\frac{\partial^3 w}{\partial x^2\partial y} \\ -(Q_{12}C+Q_{66}C)\frac{\partial^2 \emptyset}{\partial x\,\partial y}-Q_{66}C\frac{\partial^2 \psi}{\partial x^2}-Q_{22}C\frac{\partial^2 \psi}{\partial y^2} \\ +Q_{44}D\psi-\rho B\frac{\partial^3 w}{\partial y\,\partial t^2}+\rho C\frac{\partial^2 \psi}{\partial t^2} \end{matrix}\right\} = 0$$

…... *(7)*

Thus, the variationally consistent governing differential equations and boundary conditions are obtained. The values of integration constants A, B, C and D are mentioned below,

$$A=\int_{-h/2}^{h/2} z^2\,dz=\frac{h^3}{12}$$

$$B=\int_{-h/2}^{h/2} z\,f(z)\,dz=2\left(\frac{h}{\pi}\right)^3$$

$$C=\int_{-h/2}^{h/2} f^2(z)\,dz=\frac{h^3}{2\pi^2}$$

$$D=\int_{-h/2}^{h/2}\left[\frac{df(z)}{dz}\right]^2 dz=\frac{h}{2}$$

… *(8)*

The flexural behavior of the plate is described by the solution satisfying these equations and the associated boundary conditions at each edge and corner of the plate.

III. ILLUSTRATIVE EXAMPLES

Example 1: A simply supported isotropic rectangular plate subjected to uniformly distributed load. The rectangular plate occupying the region given by the (1) is considered, the plate is subjected to uniformly distributed transverse load, q(x,y) on surface z = - h/2 acting in the downward z-direction as given below:

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad \ldots(9)$$

Where, q_{mn} are the coefficients of Fourier expansion of load, which are given by,

$$q_{mn}=\frac{16q_0}{mn\pi^2}$$

for m=1,3,5,...., and n=1,3,5,....,

$q_{mn}=0$

for m=2,4,6,...., and n=2,4,6,....,

… *(10)*

The plate material are considered as *E=210 GPa and μ=0.3*, where *E* is the Young's modulus and *μ* is the Poisson's ratio. The governing differential equations and the associated boundary conditions for static flexure of rectangular plate under consideration can be obtained directly from the governing equations through given boundary conditions. The following are the boundary conditions of the simply supported isotropic plate on the edges *x=0* and *x=a*.

Navier Solution-

The following is the solution form for *w(x,y)*, *Ø(x,y)* and *ψ(x,y)* satisfying the boundary conditions given for plate with all the edges simply supported:

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\omega t$$

$$\emptyset(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \emptyset_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\omega t$$

$$\psi(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \psi_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\omega t$$

…. *(11)*

Where, *w(x,y)*, *Ø(x,y)* and *ψ(x,y)* are coefficients, which can be easily evaluated after substitution in the set of three governing differential equations (7) and solving the resulting simultaneous equations. Having obtained the values of *w(x,y)*, *Ø(x,y)* and *ψ(x,y)* one can then calculate all the displacement and stress components within the plate. *ω* is natural frequency.

IV. ANALYSIS IN ANSYS APDL

A. Modal Analysis Of Plate By ANSYS:

For the analysis of thick isotropic plate by Finite element analysis software ANSYS Model particulars:

Plate Specifications:

E= 210 GPa,
Density=7800 Kg/m^3,
Poisson's Ratio=0.3,
Length=100 mm,
Breadth=100 mm,

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Thickness=10 mm.

B. Element Type Used In Analysis:

SOLID186 is used for 3-D modeling of solid structures. SOLID186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element supports plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials.

C. ANSYS analysis has the following steps for problem solving:

Initially structural analysis is selected for the analysis of plate. After that material type is selected as isotropic, all the properties are given as per above. Modeling of 3-D plate is done by taking nodes, lines and a volume also meshing is done for the same plate. The simply supported condition is applied for given plate. Solution method is selected as modal analysis. Numbers of modes are given and solution is carried out.

V. NUMERICAL RESULTS AND DISCUSSION

Results obtained for vibrations are now be compared and discussed with corresponding results of Refined Plate Theory (RPT), Higher Order Shear Deformation Theory (HSDT) of Reddy, Classical Plate Theory (CPT) of kirchoffs, First Order Shear Deformation Theory of Ressiner and *ANSYS APDL.*

$$\%error = \frac{\text{value by perticular theory - value by ANSYS}}{\text{value by ANSYS}}$$

Results obtained for vibration are compared and discussed with the corresponding theories.

Table 1 shows comparison of non-dimensional natural bending mode frequencies of simply supported isotropic plate. It can be observed from Table that the present theory yields the good results for the frequencies for increasing frequencies. As frequency goes on increasing the corresponding error get reduced and it shows good comparison results.

Table 2 shows comparison of non-dimensional natural bending mode frequencies of simply supported plate and that of the dimensional modal frequencies obtained by ANSYS APDL. For the conversion of non-dimensional frequencies into dimensional frequency we uses the following equation.

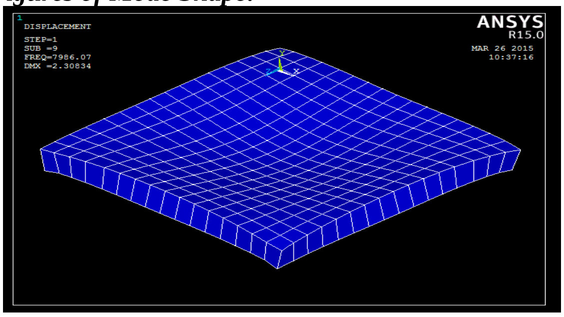
A. Result Tables:

Result tables are as shown in Appendix I

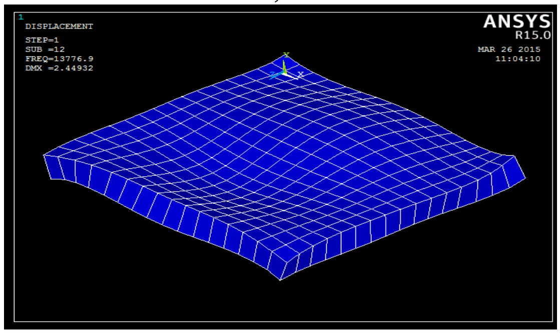
Table 1: Comparison of natural frequencies of $\bar{\omega}_{mn}$ *simply supported isotropic square plate.*

Table 2: Comparison of Natural frequencies of plate obtained byPresent theory and ANSYS program (APDL).

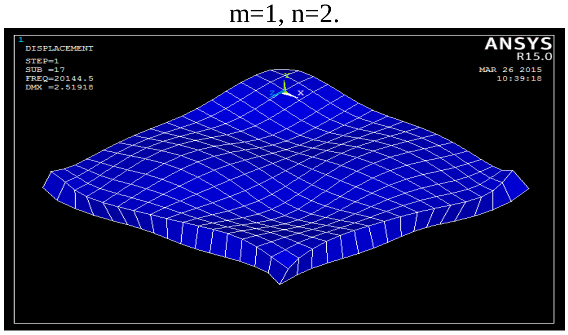
B. Figures of Mode Shape:



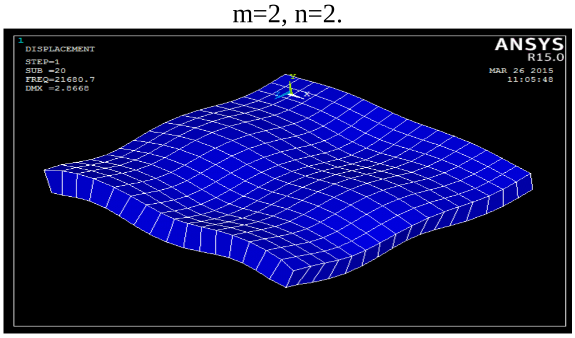
m=1, n=1.



m=1, n=2.



m=2, n=2.



m=1, n=3.

Fig. 2: Mode Shapes of Vibration in Fundamental Frequency

VI. CONCLUSION

Refined Plate Theory is most suitable theory for vibration analysis of plate by considering accuracy and Complications of calculation. In present paper Trigonometric Shear Deformation Theory is presented for isotropic plate analysis. The modal analysis in ANSYS is carried out to find out the fundamental frequencies. This theory gives the results with very less error. Also in the theory for the purpose of analysis theory removes the importance of Shear Correction Factor. From the numerical results it is concluded that the values of fundamental

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frequencies obtained by this theory are very close to the results of previously used theories and that of with ANSYS at larger values of Frequencies

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REFERENCES

[1] S.P. Timoshenko and S.W. Krieger, "Theory of Plates and Shell," McGraw Hill, New York, 1959.
[2] G.R. Kirchhoff, "Uber das gleichgewicht und die bewegung einer elastischen Scheibe, Journal für die reine und angewandte Mathematik (Crelle's Journal)," Vol.40, Pp 51-88, 1850.
[3] R. Szilard, "Theory and analysis of plates-classical and numerical methods," Prentice-Hall Inc., Englewood Cliffs, New Jersey, 2004.
[4] E. Reissner, "The Effect of transverse Shear Deformation on the Bending Elastic Plate," Transactions of the American society of Mechanical Engineers, Journal of Applied Mechanics, Vol. 12, Pp. 6977, 1945.
[5] K.H. Lo, R.M. Christensen, and E.M. Wu, "A High-Order Theory of Plate Deformation, Part-1: Homogeneous Plates," ASME Journal of Applied Mechanics, Vol. 44, Pp.663-668, 1977.
[6] K. H. Lo, R.M. Christensen and E.M. Wu, "A High-Order Theory of Plate Deformation, Part-2: Laminated Plates," ASME Journal of Applied Mechanics, Vol. 44, Pp.669-676, 1977.
[7] M. Stein, "Nonlinear Theory for Plates and Shells Including the Effects of Transverse Shearing," AIAA Journal, Vol. 24, Pp.1537-1544, 1986.
[8] J.N. Reddy, and N.D. Phan, "Stability and Vibration of isotropic, orthotropic and laminated plates according to higher order shear deformation theory," Journal of Sound and Vibration, Vol. 98, Pp. 157170, 1985.
[9] Y.M. Ghugal and A.S. Sayyad, "A static flexure of Thick Isotropic Plates Using Trigonometric Shear Deformation Theory,"Journal of Solid Mechanics, Vol.2, Pp.79-90, 2010.
[10] A.V. Krishna Murty, "Higher Order Theory for Vibrations of Thick Plates," AIAA Journal, Vol. 15, Pp.1823-1824, 1977.
[11] J.N. Reddy, "A simple Higher-order Theory for laminated CompositesPlate," Transaction of the American Society of Mechanical Engineers,Journal Applied Mechanics, Vol. 51, Pp. 745-752, 1984.
[12] S. Srinivas, C.V. Joga Rao and A.K. Rao, "Bending, Vibration and buckling simply supported thick orthotropic rectangular plates and laminates," International Journal of Solids and Structures Pp. 6:14631481, 1970.
[13] Vanam B. C. L., Rajyalakshmi M. and Inala R.' "Static analysis of an isotropic rectangular plate using finite element analysis (FEA)," Journal of Mechanical Engineering Research Vol. 4(4), pp. 148-162, April 2012.
[14] I. I. Sayyad. S. B. Chikhalthankar and Y. M. Nandedkar. "Bending and Free Vibration Analysis of Isotropic Plate Using Refined Plate Theory," Bonfring International Journal of Industrial Engineering and Management Science, Vol. 3, No. 2, June 2013, Pp. 40-46.

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APPENDIX 1

Table 1: Comparison of natural frequencies of $\bar{w}$mn simply supported isotropic square plate

(m,n)	Exact	Present	Mindlin	Reddy	Reissner	CPT	Ghugal and Sayyad
(1,1)	0.0932	0.09322	0.093	0.0931	0.093	0.0955	0.0933
(1,2)	0.2226	0.2224	0.2219	0.2222	0.2219	0.236	0.2231
(2,2)	0.3421	0.3404	0.3406	0.3411	0.3406	0.3732	0.3431
(1,3)	0.4171	0.4161	0.4149	0.4158	0.4149	0.4629	0.4184
(2,3)	0.5239	0.5229	0.5206	0.5221	0.5206	0.5951	0.5258

Table 2: Comparison of Natural frequencies of plate obtained by Present theory and ANSYS program (APDL).

(m,n)	By TSDT (Dimensionless)	By formula(11)	By ANSYS APDL	Error in %
(1,1)	0.09322	5017.85	7986	36
(1,2)	0.2224	12129.62	13776.9	11.95
(2,2)	0.3404	18662.2	20144.5	7.35
(1,3)	0.4161	22757.99	21680.7	4.96
(2,3)	0.5229	28599.99	27654	3.42

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