Protracted Network: Quality of Service

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Abstract—The capability of a system to continuously deliver services in compliance with the given requirements in the presence of failures and other undersigned events, is a property of protracted network. An easy solution to provide good quality of service is to build a network with enough capacity. A strong network should have a important property that the network should be designed in such away that it must take no time or very small time to recover from a big disaster. The objective of this paper is to provide an overview of network connectivity in relation to network protection design. In this paper we aim to introduce and analyze the advantages and disadvantages of methods and algorithms for searching good network connectivity as well as sets of disjoint and distinct paths for protection design. Here we will make 2-connected network to improve network performance.

Keywords: Graph theory, network connectivity, survivability

I. INTRODUCTION

Networks are everywhere. A network, which we can informally define as large collection of interconnected nodes. A node can be anything: a person, an organization, a computer, a biological cell, and so forth. Interconnected means that two nodes may be linked, for example, because two people know each other, two organizations exchange goods, two computers have a cable connecting the two of them, or because two neurons are connected by means of a synapses for passing signals. And a network is said to be **protracted** if all of the demands can be met under the failure of any one of its links. Apart from the lot of traffic, there can be severe consequences when a physical link fails. Network failure which may be caused by dig-ups, vehicle crashes, human-errors ,system malfunctions, fire ,rodents ,sabotage, natural disasters(e.g. flood, earthquakes, lightning, storms), and some other factors have occurred quite frequently and sometimes with unpredictable consequence .An easy solution to provide good quality of service is to build a network with enough capacity for whatever traffic will be thrown at it. But the trouble with this solution is that it is expensive. It is basically solving a problem by throwing money on it. Quality of service mechanisms ensures how to reserve resources to guarantee performance. In this paper, we will use the term *protracted networks* to refer to networks that, when a component fails, may "alive" by finding alternative paths that circumvent the failed component. Three ingredients are needed to make a protracted network:

***(1)Strong network connectivity*:** There we mean that network should be as invulnerable to destruction of individual stations and individual lines as possible.

(2) Network augmentation: , that is, new links may need to be added to strengthen the connectivity of a network.

(3) Path cover: is a procedure to find alternative paths in case of failures. Above mentioned points will be explained in the following sections.

***A . Connectivity*:** In all the graphs we have considered so far, each vertex 'v' could be reached from any other vertex 'w' in the sense that we could indicate a chain of adjacent vertices from 'v' to 'w'. In this section, we will take a closer look at this important concept of connectivity. A network is often represented as a graph G (V,E), where V is the set of nodes (which for instance represent routers or stations) and E is the set of L links (which for instance represent optical fiber lines or radio channels or simply edges). Links may be characterized by weights representing for instance their capacity, delay, length, cost, and/or failure probability. A graph is said to be connected if there exists a path between each pair of nodes in the graph, else the graph is said to be disconnected. In other words, the physical topology must remain connected under the failure scenario. For example, to cope with single link failures in the network, the physical topology must be at least 2-connected, meaning that there is at least 2 link-disjoint paths between any two nodes in the network. Generally, to protect against the failure of any set of k links in a network, the physical topology of that network must be $(k+1)$-connected. Depending on whether these paths are node or link disjoint, we may discriminate between node and link connectivity. The link connectivity $\lambda(G)$ of a graph G is the smallest number of links whose removal disconnects G. Correspondingly, the node connectivity $\kappa(G)$ of a graph is the smallest number of nodes whose removal disconnects G. In 1927, Menger provided a theorem [1]—in German— that could be interpreted as follows:

Theorem 1 (Menger's theorem). *The maximum number of link /node-disjoint s-t paths is equal to the minimum number of links/nodes whose removal disconnects t from s.*

1. Suppose the removal of $F \subseteq E$ disconnects t from s, and $|F|$ = k

2. All s-t paths use at least one edge of F. hence the number of edge disjoint paths is at most k.

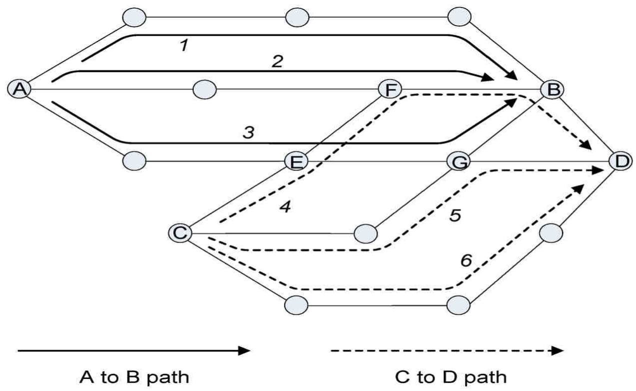

Fig. 1: Maximum edge disjoint path between two vertices

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The notion of connectivity is important, notably when considering the robustness of networks. Robustness in this context means how well the network stays connected when we remove vertices or edges. For example, as we know that the Internet can be viewed as a (huge) graph in which routers form the vertices and communication links between routers the edges. In a formal sense, the Internet is connected. However, if it were possible to partition the network into multiple components by removing only a single vertex (i.e., router) or edge (i.e., communication link), we could hardly claim the internet to be robust. In fact, it is extremely important for networks such as the internet to be able to sustain serious attacks and failures by which routers and links are brought down, such that connectivity is still guaranteed.

B. Network Augmentation**:** All customer use capacity in the existing network proportional to their demand. In some cases the network requires immediate upgrading, and in other cases, this happens later on. The goal of this improvement process is to maximize the connectivity of the networks . One option for improving a network configuration is by introducing additional link resources from a connectivity standpoint.

C. Path cover: A physical topology is considered to be protracted if it can cope with any single failure of network components by rerouting those connections affected by the failures through alternative paths. Clearly, this requires some resource redundancies in the network. So we provide the ability to switch over in subsequent time from a failed primary path to an alternate path. This alternate path can either be configured to protect against a link or node failure. Using the graph theory terminologies, a protracted network must be at-least 2-connected or a biconnected graph, meaning that there is at least 2-link disjoint paths between any two nodes in the network.

II. GRAPH THEORETIC APPROACH TO ESTABLISH NETWORK PROTECTION DESIGN

In Section I. A, we indicated that Menger's theorem implies that finding a minimum cut corresponds to finding the connectivity of a network. In this section, we will look further at finding cuts in a network.

Definitions:

1. *Edge or link cut*: A link cut refers to a set of links whose removal separates the graph into two disjoint sub graphs , and where all links in the removed cut-set have an end-point in both the sub graphs. The two sub graphs need not be connected themselves.

2. Vertex or node cut: A node cut refers to a set of nodes whose removal separates the graph into two disjoint sub graphs, and where all nodes in the removed cut-set have at least one adjacent link to both sub graphs.

3. Minimum link/node cut: A minimum cut is a cut whose cardinality is not larger than that of any other cut in the network. Definitions for a cut also have a variant in which a source node s and a terminating node t need to be separated.

4. s-t cut: An s-t cut refers to a cut that separates two nodes s and t in the graph such that both belong to different sub graphs. Often, when referring to a cut, a link cut is meant. In the remainder of this paper, we will use the same convention and only specify the type of cut for node cuts.

5. Maximum cut: A maximum cut is a cut whose cardinality is not exceeded by that of any other cut in the network. Practically, different traffic requirements over a network would require different connectivity between nodes. Some traffic demands may require no protection, or may only need to be carried when possible. In contrast, other traffic demands may ask for a full protection against either single-link failure, dual-link failures or other types of failure. Hence, the required connectivity between nodes would be different for these varied protection requirements. The service quality of a network can be maintained by designing the network in such a way that it works under network failure. Deterministic techniques usually involve the evaluation of certain graph theoretic concepts associated with the network. Boesch & Frisch [2] defined a measure of network vulnerability, viz, the number of elements in the smallest cut-vertex set. The vulnerability measures relation of the connectivity level of a network to the number of node and/or link disjoint paths between node pairs in the network. Node disjoint paths have no common nodes except the source and destination nodes, while link disjoint paths contain no common links. Instead, it will consider the set of specific (or predetermined) failures. For example, protection design against single link failures needs to determine the recovery routes for services so that they can maintain their services under the failure of any single link in the network. To do that, the network connectivity must provide at least ***2 disjoint paths*** between any source and destination nodes. The term "disjoint" here is with respect to the failure scenario, meaning that the "disjoint" paths must not suffer from the same failure. For example, for single link failures, the 2 paths must be link-disjoint. Similarly, for single node failures, the 2 paths must be node-disjoint. This section presents the general requirement for network connectivity in which the network can at least be protected against any single failure scenario, e.g. the failure of any single link or node of the network. In this case, the physical topology of the network must be 2-connected. Further we know that a graph is connected if for any two vertices $x, y \in V(G)$, there is a path whose endpoints are x and y. A connected graph G is called 2-connected, if for every vertex $x \in V(G)$, $G - x$ is connected. Also it is important to notice that , node-disjoint paths are always link-disjoint. A graph which provides at least 2 link-disjoint paths between any two nodes is 2-*connected*. With stronger connectivity, a *bi connected* graph is able to provide at least 2 node-disjoint paths between any two nodes. Generally, a network must provide at least K link-disjoint paths between any node-pairs to be able to protect against simultaneous failure of $K - 1$ links. The graph of such networks is said to be ***K-connected*****.** Establishing the physical topology of a protracted large networks is not a trivial task. Some techniques for assessing physical survivability such as the *cut set* method can not deal with large size networks [1], [2]. A fast technique for finding *bi connected components* of a graph and testing the network for node-/link-bridges, presented in [3], does not provide any further information, such as identifying the fundamental cycles within the network. This paper presents an alternative technique, based on graph

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theory, for evaluating the physical survivability of networks. This technique can deal with network sizes of many thousand nodes, with computational times which are comparable with the *bi connected components* method, while providing more information about the susceptibility of a network to individual link and node failures. So, this paper will mainly discuss network connectivity which supports single link and single node failures, the minimum requirement, known as 2-connected and bi connected.

III. NETWORK DESIGN

This section presents the procedures for designing a protracted network against single-link failures which is one of the most common failure scenarios occurring in practical networks. As discussed, the physical topology of a network can only be protracted under single-link failures if and only if it is 2-connected. The concept of protracted networks is more complex than the concept of connectivity in graph theory. In addition, efficient automation algorithms based on graph theory can help designers to reduce the computational time and avoid human errors. This section presents techniques for evaluating the physical topology for protracted networks. Firstly, we outline and analyze the strengths and weaknesses of a popular method, namely the cut-set method. Then, we introduce technique that can deal with network sizes of many thousand nodes [4] , which uses properties of 2-connected graphs.

A. Protraction through cut-sets

A network is protracted if the size of every cut-set of the network is equal to or larger than 2. At a glance, this definition leads to a view that the network has nodal-degree of two, meaning that every node in the network is connected to at least two other nodes. Since every node is connected to at least two other nodes in the network, on the surface this property seems to be able to offer two disjoint paths between any two nodes in the network. In fact, this is a misconception. If a network is 2-connected then the nodal degree of all nodes in the network is equal to or larger than 2. The reverse does not hold, however, in that a network in which the nodal degree of all its nodes equal to or larger than 2 is not always 2-connected. The topology in Fig. 2 illustrates this concept.

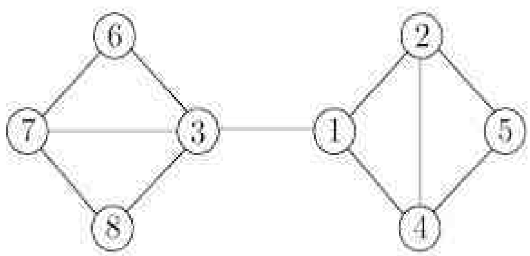

Fig. 2: Failure of nodal degree technique

In the figure we can see that path (3 − 1). It is a bridge that connects two subsets of network nodes $X = \{3,6,7,8\}$ and $Y = \{1,2,4,5\}$. As a result, all paths between nodes $x \in X$ and $y \in Y$ must share the same path (3 − 1). Hence, although all nodes in this network have a nodal degree equal to or larger than 2, it is not a 2-connected network. Therefore, all algorithms for verification of network survivability based on node-degree of two may yield undesirable and inaccurate results and hence are not reliable. The cut-set assumption described below has been preferred for the accuracy of network survivability verification. Let $G = (V, E)$ be a network topology. A cut in G is a partition of V into parts S and $\bar{S} = V \setminus S$. Each cut defines a set of edges consisting of those edges in E with one end-point in S and the other in $\bar{S}$. This edge set is referred as the cut-set $CS(S, V \setminus S)$ associated with the cut $\langle S, V \setminus S \rangle$. Let $|CS(S,V \setminus S)|$ be the size of the cut-set, being the number of links between S and $V \setminus S$. Thus, according to the cut-set assumption, a network is 2-connected, **if** $|CS(S,V \setminus S)| \geq 2$, $\forall S \subset V$. If S is a subset of only a single node in the network, then the cut-set assumption is essentially the same as the node-degree assumption. Since the cut-set assumption is related the number of links connected between two subsets of a cut, it can assure the network to offer ***link-disjoint paths****, but* ***not node-disjoint paths****.* Here we have a related theorem by Menger which determines the connectivity of a network by examining its cut sets.

Theorem 2: A topology with the set of vertices (nodes)N and the set of edges (links) E is 2-connected if and only if every non-trivial cut $\langle S, V \setminus S \rangle$ has a corresponding cutest of size greater than or equal to 2.

Proof: In other words, a configuration of the network that satisfies the condition of the cut-set assumption can provide at least one link-disjoint path-pair between any distinct pair of source node and destination node. The implementation of the cut-set assumption is not complex but its computational time for large scale networks is its biggest disadvantage. The number of cut-sets increases exponentially with the number of network nodes and is calculated as in [3]:

$$N_{cut\ set} = 2^{|V|} - 2$$

where $N_{cut\ set}$ is the number of cut-sets in the network, and $|V|$ is the number of nodes. The number of cut-sets doubles with an increase of one node in the network. For instance, $N_{cut\ set}$ in a network of 20 nodes is over 1 million; and it is over 32 million with $|V| = 25$; which is $32(= 2^5)$ times larger than $|V| = 20$; and the number of cut-sets in the networks of $|V| = 30$ nodes is up to 1 billion cut-sets. So, the cut-set technique becomes intractable even with moderate scale networks ($20 \leq |V| \leq 30$). In summary, the node-degree assumption is simple but not reliable for the verification of network survivability. Meanwhile, the cut-set assumption is only applicable for link-survivable networks, and it is intractable with large scale networks. The node-degree assumption cannot verify any type of physical topology that has potential to support protracted network (namely 2-connected and bi connected networks) whereas the cut-set assumption can verify the survivability of a network that is 2-connected but cannot identify exactly a 2-connected topology or verify a bi connected topology. Next, we propose an approach that can classify network topologies, and determine if they are unconnected, (1−)connected, 2-connected or bi connected.

A. Protracted through 2-connected

From Theorem 2, it can be deduced that the cut sets of a cycle always have a size of 2. Furthermore, a 2-connected graph can be easily constructed from simple cycles [5]. The following proposition implies a method for constructing such graph.

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Proposition 1: A graph is 2-connected if and only if it can be constructed from a cycle by successively adding H-paths to graph H already constructed.

Proof: Cleary, every graph constructed as proposed is 2-connected. Conversely, let G be a 2-connected graph, then G contains a cycle, and a sub graph H is constructible, as evident in Fig 3.

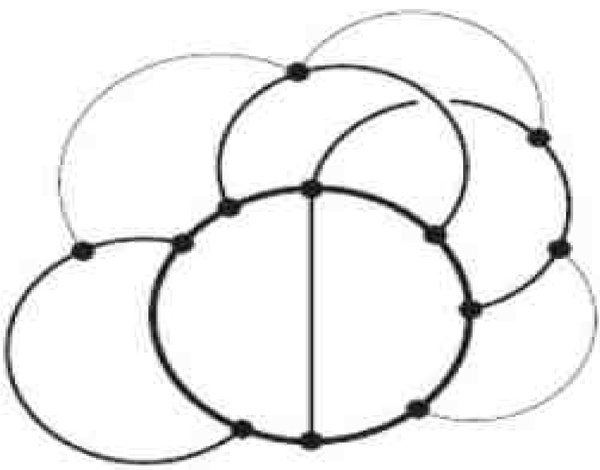


Fig. 3: Construction of 2-connected graph

Any edge $x, y \in E(G)\backslash E(H)$ with $x, y \in H$ defines a H-path. Then, H is an induced sub-graph of G. If $H \neq G$, then by the connectedness of G, there is an edge vw with $v \in G - H$ and $w \in H$. As G is 2-connected, $G - w$ has a $v - H$ path P. Then wvP is a H-path in G, and $H \cup wvP$ is a constructible sub-graph of G.

IV. AN APPROACH TO CLASSIFY NETWORK TOPOLOGIES

Assume that G' and G'' are two blocks of graph G. From Proposition 1, we can deduce the connectivity of graph G depending on the relation between G' and G'', as described below:

1) If G' and G'' have at least 2 common vertices, then G is a 2-connected graph with no cut vertex (i.e. node bridge) or cut edge (i.e. link bridge), i.e. G is a bi connected graph.

2) If G' and G'' only have one common vertex, then G is a 2-connected graph with a cut vertex which is the common vertex.

3) If G' and G'' are separated by a cut edge, then G is not a 2-connected graph, and the cut edge cannot be protected.

4) If G' and G'' have no common links or nodes, then G is not a 2-connected graph, and therefore it is not survivable.

Based on the above discussion, we can use the relationship between networks' cycles or 2-connected graphs to verify the survivability of its physical topology. An undirected graph is thus seen as the combination of all the fundamental cycles. Using Alg. 1, these fundamental cycles can be found from a spanning tree $\{V, T\}$ of a graph $G = \{V,E\}$ (eg. the spanning tree highlighted by thick lines in Fig. 4).

Algorithm 1 *Finding cycle*

Input : A tree T and an edge e whose end-nodes is in T;

Output: A cycle P formed by T and e;

init

$(s, d) \leftarrow$ end-nodes of e;

$queue \leftarrow [node.s, node.P]$; $check \leftarrow 0$;

while $check == 0 \& queue \neq \Phi$ **do**

$[v] \leftarrow head(queue)$; $queue \leftarrow queue - \{head(queue)\}$;

if $v.s == d$ **then**

$check = 1$; $P \leftarrow v.P$

else

for all v_k is neighbour of $v.s$; **do**

$node.s \leftarrow v_k$; $node.P \leftarrow P \cup v_k$;

push *node* into *queue*;

end for

end if

end while

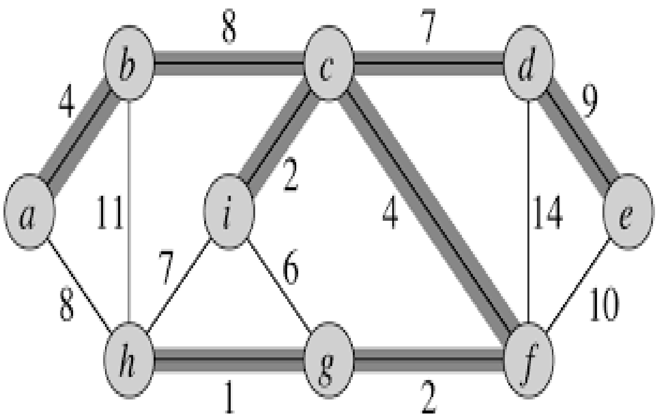
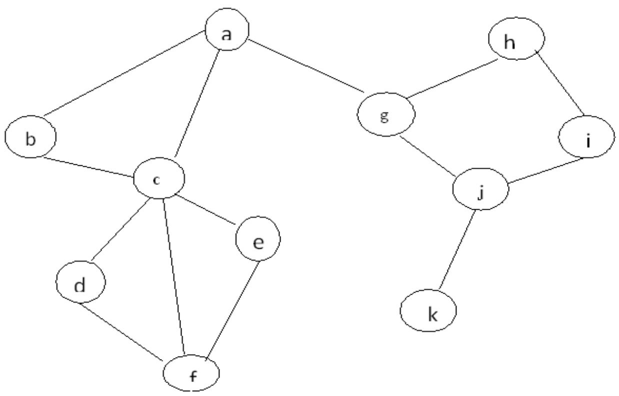
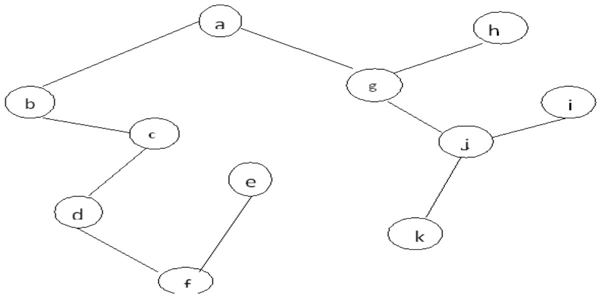


Fig. 4: Spanning tree (With thick lines) in a graph G

Any set of cycles found from the spanning tree can be used to verify how well the network is protracted.. An algorithm for finding a set of cycles through spanning treeof a graph is represented in Alg. 1. An efficient method for finding fundamental cycles of a graph, referred to as *Paton's* algorithm, is outlined in [6]. If a graph is 2-connected, then each vertex of the graph will be at least on one of the cycles resulting from Alg. 1. Hence, such set of cycles is sufficient to verify that how much this network is protracted.



(a) An arbitrary physical structure



(b) The spanning tree

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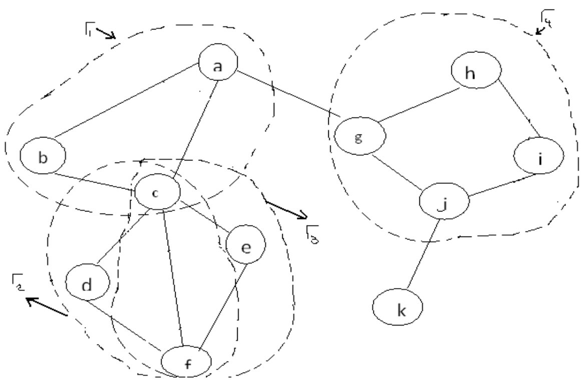

(c) Protracted network outcomes

Fig. 5: An illustrative example

Here, we shall give an example of how our approach works over an arbitrary physical topology *G* as shown in Fig. 5(a), with the set of nodes *V* and edges *E*. Since this topology is an connected topology, the first step results in a tree *T*, being a sub graph of *G*, and , as shown in Fig. 5(b). *T* has a set of nodes *VT* and edges *ET* , where *VT* = *V* , and *ET* = *E* − {(a, c), (c, e), (c, f), (h, i)}. The spanning tree can be determined using Prim's algorithm or Kruskal's algorithm. Next, a set of cycles is found using Alg. 1. In our example, this consists of 4 cycles $\{\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4\}$ as shown in Fig. 5(c). The input of the second step is the spanning tree *T* of Fig. 5(b), and the output is shown in Fig. 5(c). Note that topology *G* contains 3 maximal survivable-bases, namely as $S1 = \{\Gamma_1\}$ $S2 = \{\Gamma_2, \Gamma_3\}$, and $S3 = \{\Gamma_4\}$. *S*1 and *S*2 share node 'c' in graph *G*, hence node 'c' is a *cutvertex* (or node-bridge). There are 2 link-bridges which are (a − g) and , (j − k) . Node *'k'* which is not part of any 2-connected block is referred to as single node.

V. CONCLUSION

In this paper, we have discussed the connectivity of the physical topology to support the problem of designing multiple quality of protections. Here we have presented an approach for evaluating the physical topology of large protracted networks. The computational efficiency of this approach, when dealing with large networks, is comparable to the *bi connected components* approach in [3]. This technique is also capable of providing all the distinct fundamental cycles of the network, if required. 2-connected graph theorem can be used to identify the weak nodes/links of a given large size network much faster than some other techniques such as 'cut set'. Furthermore, it also provides information about all distinct cycles in the network, useful for the next phase of network planning, which cannot be provided by any other *technique*.

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