State Feedback Sliding Mode Control for an Active Magnetic Bearing System

Yao-Wen Tsai, Van Duc Phan

Abstract—This paper proposes the state feedback Sliding Mode Control (SMC) approach in order to control the nonlinear system. A nonlinear model of two degrees of freedom (DOF) of an Active Magnetic Bearing(AMBs) obtained using Lagrange's equation is introduced. The SMC approach by using linear matrix inequality (LMI) technique is proposed not only to out-perform the proportional integral differential (PID) control but also to show some advantages. Firstly, a robust stabilization problem for a class of nonlinear systems is considered. Secondly, the conservatism of PID approach is reduced, fast response and reject disturbance of the system is also enhanced in this study. Finally, the simulation result has been obtained and compared with the conventional PID control. website.

Keywords—state feedback control, two DOF for AMB, sliding mode control.

I. INTRODUCTION

AMBs have been successfully used in various applications for several decades. They show great abilities to work under extreme conditions, such as vacuum, high rotation speed or at high temperature, enable non-contact operation and can guarantee a good performance of the system at high speed without lubrication [1]-[2], [4]-[7]. However, modeling and control of AMBs have still been challenging problems, since AMBs have unstable behavior and are nonlinear mechatronic systems. Most of the control design approaches for AMBs are based on the linearized model about a nominal operating point. The behavior of the linear model is acceptable when the operating point is close enough to the linearized point [4]-[8]. In order to ensure the system's performance in a wide range of working conditions, a nonlinear model should be considered in controller design.

In modern industrial engineering, SMC has become the most popular strategy to control practical systems. Moreover, SMCs have many advanced control techniques that use a dynamic model of the system to give stability, fast response and robustness [4]-[6], but all of them are based on the linearized model. In this paper, two degrees of freedom (DOF) is introduced. A nonlinear electromechanical model of this system is also derived from Lagrange's equation by using symbolic computation package such as Maple®. The SMC approach is presented and a control strategy is applied to regulate the nonlinear system. In addition, the sliding surface is designed in term of LMIs to guarantee the stability of system dynamics in sliding mode.

Finally, numerical simulation results are presented to demonstrate the dynamic behavior of the system, and the performance of SMC for this machine is compared with classical PID control.Use italics for emphasis; do not underline.

II. MATHEMATICAL ANALYSIS

A. *Electromechanical model*

In this section, a model of AMB with a single mechanical degree of freedom Fig. 2.1 is introduced to illustrate the Lagrange's equation approach for an electromechanical system [3].

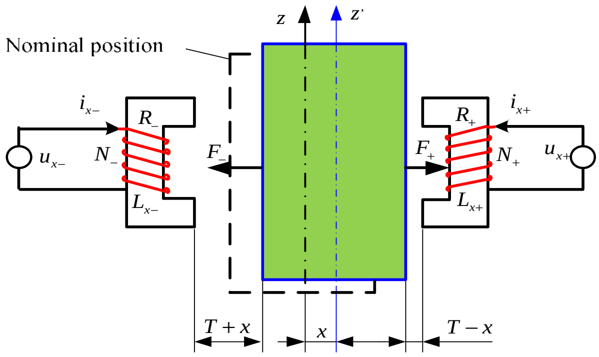

Fig. 2.1 Single DOF of AMB

$$
\begin{aligned}
Ke_M &= \frac{1}{2} m\dot{x}^2, \; V_M = 0, \\
Ke_E &= \frac{1}{2} L_{x+}\dot{q}_{x+}^2 + \frac{1}{2} L_{x-}\dot{q}_{x-}^2, \; V_E = 0.
\end{aligned} \tag{1}
$$

Energy contributions of this system are showed in Equation (1).

Where:

- Ke_M , V_M are the kinetic and potential energy of mechanical part.
- Ke_E and V_E are the kinetic and potential energy of electrical part.
- The electrical charge in each coil, q_{x+}, q_{x-} is generalized coordinates of electrical part.
- x is the displacement of the rotor.
- L_{x+}, L_{x-} are coil inductances.

The relation of coil inductance with air gap T and the coil characterizing parameters is described in Equation (2).

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$$L_{x+} = \mu_0 \frac{N^2 A}{2(T-x)}, \quad L_{x-} = \mu_0 \frac{N^2 A}{2(T+x)}. \tag{2}$$

Where: μ_0, T, A, N, R, K: Air permeability, nominal air gap, cross section area, number of coils, coil resistance and sensor gain.
The dissipation of copper losses in the coils is

$$P = \frac{1}{2} R\dot{q}_{x+}^2 + \frac{1}{2} R\dot{q}_{x-}^2 \tag{3}$$

The dynamic equation of single DOF AMB model can be derived from Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial \dot{s}}\right) - \frac{\partial \mathrm{L}}{\partial s} + \frac{\partial P}{\partial \dot{s}} = Q \tag{4}$$

Where s is the generalized coordinate vector

$$s = \left[q_{x+}, q_{x-}, q_{y+}, q_{y-}, \dot{x}, x\right]^T, \tag{5}$$

Q is a vector of generalized external forces (control input voltage and mechanical force)

$$\mathrm{Q} = \left[u_{x+}, u_{x-}, u_{y+}, u_{y-}, F_x, 0\right]^T, \tag{6}$$

and L is the Lagrangian function

$$\mathrm{L} = Ke_M + Ke_E - V_M - V_E \tag{7}$$

B. *Two DoF of the AMB*

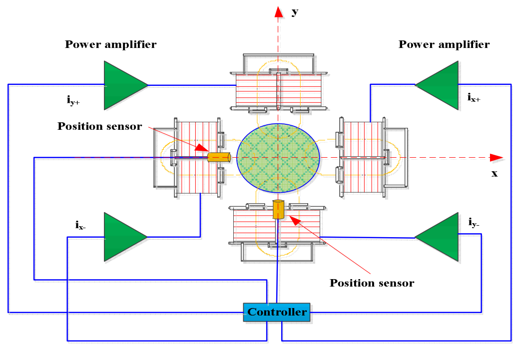

Fig. 2.2 Two DOFs model for AMBs

We applied the Equation (4) and (7), the Equation of motion of the system can be derived in a standard nonlinear form of differential Equation as

$$K\dot{s}_{state} = J \tag{8}$$

where

$s_{state} = \left[i_1\ i_2\ i_3\ i_4\ \dot{x}\ x\ \dot{y}\ y\right]^T$ and

$J = \left[u_1\ u_2\ u_3\ u_4\ F_x\ 0\ F_y\ 0\right]^T$

and $K \in R^{8x8}$ is the inertial matrix and $J \in R^{8x1}$ is the vector of nonlinear function. These Equation are solved in Jacobian by using Maple 17 software.

C. *Parameters of two DoF of the AMB*

The physical parameters of this two DOF of AMB model for simulation are given follow tables.

Table 1: Rotor parameters

Parameters	Symbol	Value	Unit
Mass	m	5.31	Kg
Bias current	i_0	0.45	A
Resistor of coils	R	0.225	Ω
Air permeability	μ_0	4πe-3	H/m
Coil resistance	N	150	m

III. Control Design and Simulation Result

A. *Sliding surface design*

In this section, two degrees of freedom of the AMB is controlled via SMC approach. The type of model being considered in this section is discrete and linear time-invariant (LTI) the Equation (8)and the state-space form given by

$$\dot{s}_{state} = As_{state} + Bu + Df \tag{9}$$

Where $A \in R^{8x8}$, $B \in R^{8x4}$ are system matries and is disturbance matrix $D \in R^{8x4}$ with $f = [\sin \pi t + \cos 2t]$. With any $\tilde{B}$ basis of the null space of B^T, i.e. $\tilde{B}$ is an orthogonal complement of B. Consider the following linear matrix inequalities (LMIs):

$$X > 0, \quad \tilde{B}^T\left(AX + XA^T\right)\tilde{B} < 0. \tag{10}$$

The linear sliding surface σ is given by the following explicit formula:

$$\sigma = Ss_{state} = B^T X^{-1} s_{state} = 0 \tag{11}$$

Now, let the control law be given as follows:

$$u = -(SB)^{-1}\left(\frac{k_1}{2} + k_2\|x\| + k_3 h\right)\frac{\sigma}{\|\sigma\| + \varepsilon} \tag{12}$$

where ε is any positive scalar and $k_1 > 0,\ k_2 > \|SA\|,\ k_3 > \|SD\|$

Theorem 1. *Suppose that the LMIs* (10) *has a solution* X *and the linear sliding surface is given by Equation* (11)*. And consider the system* (9) *with control* (12)*. Then, the reduced-order system dynamics restricted to the switching surface* $\sigma = 0$ *is asymptotically stable. And every solution trajectory is directed towards the linear switching surface and remains on the surface for all subsequent time.*

Proof. Define a transformation matrix and the associated vector v as follows:

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$$M=\begin{bmatrix}\tilde{B}^T\\ B^T X^{-1}\end{bmatrix}=\begin{bmatrix}\tilde{B}^T\\ S\end{bmatrix},\ v=\begin{bmatrix}v_1\\ v_2\end{bmatrix}=Ms_{state},$$

where $v_1\in R^{2x1}$, $v_2\in R^{6x1}$ and $v_2=\sigma$. We can see that $M^{-1}=\left[X\tilde{B}\left(\tilde{B}^T X\tilde{B}\right)^{-1}\ \ B(SB)^{-1}\right]$. By the transformation on can obtain

$$\dot{v}=\hat{A}v+\hat{B}u \tag{13}$$

where

$$\hat{A}=MAM^{-1}=\begin{bmatrix}\tilde{B}^T AX\tilde{B}\left(\tilde{B}^T X\tilde{B}\right)^{-1} & \tilde{B}^T AB(SB)^{-1}\\ B^T X^{-1}AX\tilde{B}\left(\tilde{B}^T X\tilde{B}\right)^{-1} & B^T X^{-1}AB(SB)^{-1}\end{bmatrix},$$

$\hat{B}=MB=\begin{bmatrix}0\\ B^T X^{-1}B\end{bmatrix}$. The system (13) in the sliding mode $\dot{\sigma}=\sigma=0$ is governed by the following system dynamics:

$$\dot{v}_1=\tilde{B}^T AX\tilde{B}\left(\tilde{B}^T X\tilde{B}\right)^{-1}v_1 \tag{14}$$

The following Lemma will be used

Lemma 1 *[9]. Consider the following uncertain system*

$$\dot{x}(t)=Ax(t) \tag{15}$$

The system (15) *is said to be quadratically stable if there exists a positive definite symmetric matrix* P *such that*

$$AP+PA^T<0. \tag{16}$$

By Lemma 1, one can see that the reduced-order equivalent system (14) is quadratically stable if there exists a positive-definite matrix P such that

$$\tilde{B}^T AX\tilde{B}\left(\tilde{B}^T X\tilde{B}\right)^{-1}P+P\left(\tilde{B}^T X\tilde{B}\right)^{-1}\tilde{B}^T XA^T\tilde{B}<0 \tag{17}$$

Choosing $P=\tilde{B}^T X\tilde{B}$ and substituting it into the above inequality yield

$$\tilde{B}^T\left(AX+XA^T\right)\tilde{B}<0 \tag{18}$$

Obviously, the inequality (18) is equivalent to (10). Using the linear sliding surface, one can obtain

$\dot{\sigma}=SAs_{state}+SBu+SDf$. Let Lyapunov function

$$V=\sigma^T\sigma \tag{19}$$

If we differentiate (19), with respect to time combined with (11), and (9) then we have:

$$\begin{aligned}
\Rightarrow \dot{V}&=2\sigma^T\dot{\sigma}=2\sigma^T S\dot{s}_{state}\\
&=2\sigma^T S(Ax+Bu+Df)\\
&\le 2\|\sigma\|\|SA\|\|x\|+2\sigma^T S(Bu+Df)\\
&\le 2\|\sigma\|\|SA\|\|x\|+2\|\sigma\|\|SD\|h+2\sigma^T SBu\\
&=2\|\sigma\|\|SA\|\|x\|+2\|\sigma\|\|SD\|h\\
&\quad-2\sigma^T SB(SB)^{-1}\left(\frac{k_1}{2}+k_2\|x\|+k_3h\right)\frac{\sigma}{\|\sigma\|+\varepsilon}\\
&=2\|\sigma\|\|SA\|\|x\|+2\|\sigma\|\|SD\|h\\
&\quad-2\sigma^T\left(\frac{k_1}{2}+k_2\|x\|+k_3h\right)\frac{\sigma}{\|\sigma\|+\varepsilon}\\
&=-k_1\|\sigma\|<0.
\end{aligned}$$

This completes the proof.

B. Simulation

In this section, dynamic behaviors of the system and control performance are discussed in simulation result by using Maple to solve the Equation (8).

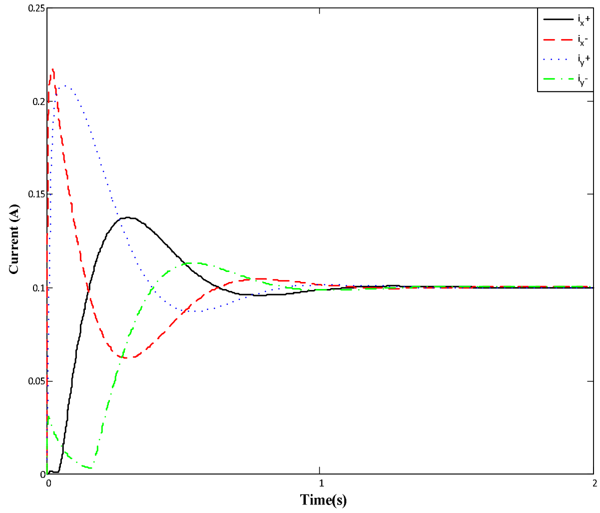

Fig. 3.2-1 Time respone of the currents for each coil of PID control.

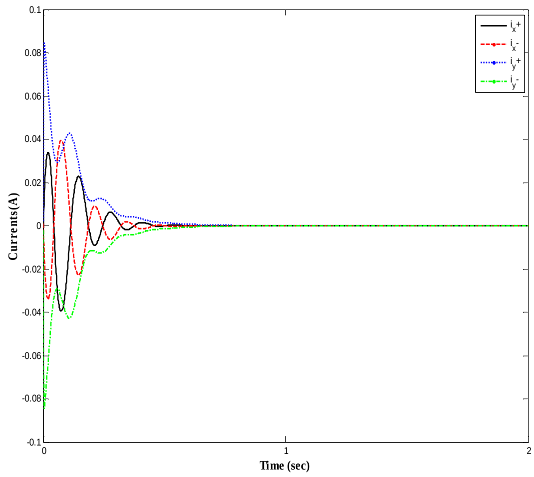

Fig. 3.2-2 Time respone of the currents for each coil of SMC control.

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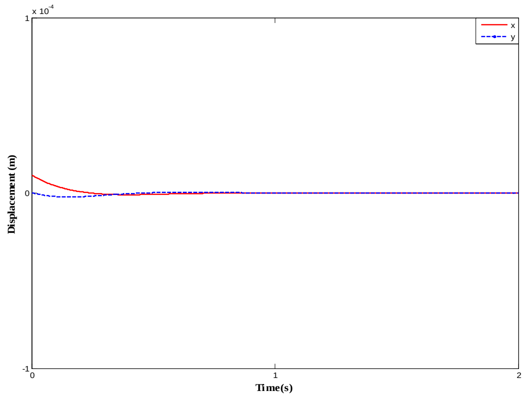

Fig. 3.2-3 Time respone of the displacements for each coil of PID control.

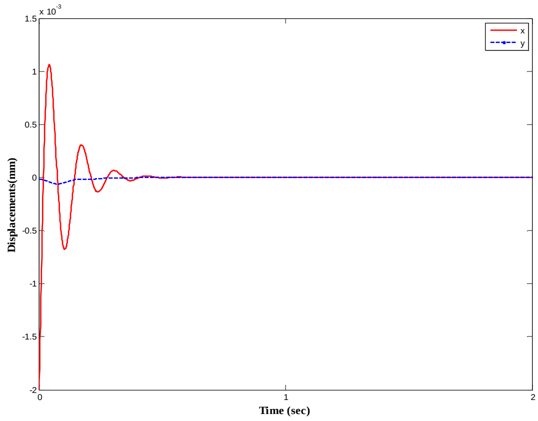

Fig. 3.2-4 Time respone of the displacements for each coil of SMC control

From Fig. 3.2-1 to Fig. 3.2-4, it is easy to see that the proposed controller of SMC has a good performance, fast response than PID and is effective in dealing with the disturbance.

IV. Conclusion

In this paper, the nonlinear system model, the so-called are active magnetic bearings for a two DOF, is introduced. The system a structure of an active magnetic bearing two DOF is obtained by Lagrange's equation. In this model, the current in each coil is treated as a state variable and the control input is the voltage applied to each coil, this approach offers more advantages than current control input approach. It is more reality and also allows us to synthesize the controllers with the control input is voltage. Dynamic behavior of the two DOF in magnetic bearings and performance of the controller of the SMC approach are superior to PID. In addition, the sliding surface is designed in term of LMIs to guarantee the stability of system dynamics in sliding mode. SMC technique has shown that the controller can guarantee the rotor stay in the center even disturbance effect.

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